Tripoli university

Faculty of engineering

EE department

EE313 tutorial

Problem#1

Find the flux of the vector field $\vec{A} = \frac{\vec{a}_\rho}{\rho}$ for:

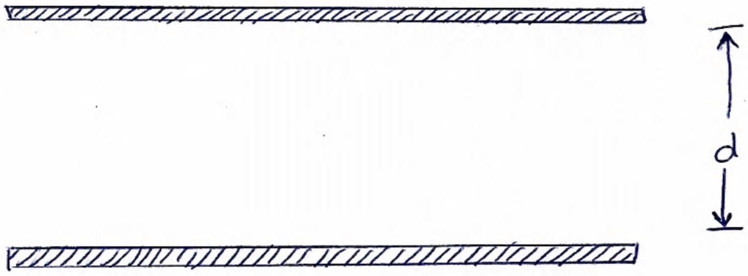
i) The sphere r=a centered at the origin.

ii) The cube 2a on a side centered at the origin with sides parallel to the coordinate axes.

iii) The cylinder $0 \le \rho \le 3a\,, 0 \le \phi \le 2\pi\,, -a \le z \le a\,.$

Problem#2

For the parallel plates shown in the fig at time t=0 an electron is emitted from the lower plate with zero initial velocity and upper plate is at 15V higher than the lower. At time t_1 the electron is midway between the plates and the upper plate voltage changes abruptly to -30V. Determine which plate the electron will strike.

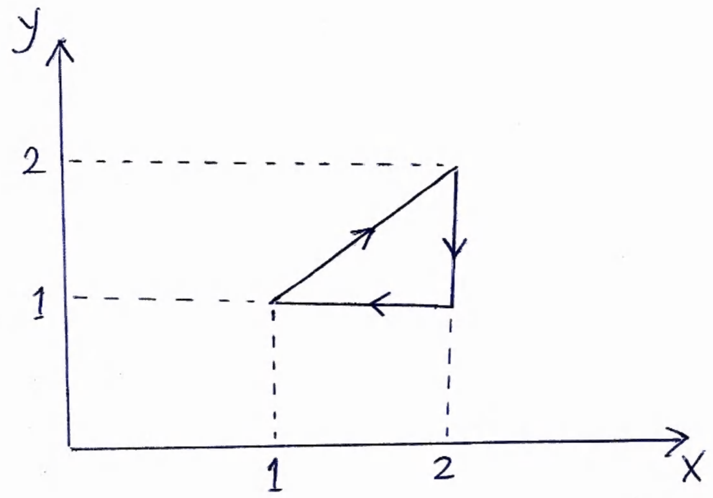


Problem#3

Assume the vector function $\vec{A} = \vec{a}_x 3x^2y^3 - \vec{a}_y x^3y^2$.

i) Verify Stoke's theorem for the surface shown in the fig.

ii) Can $\vec{A}$ be expressed as the gradient of a scalar? Explain.



Problem#4

A flat slab of sulfur ($\epsilon_r = 4$) is placed normal to a uniform field. If the polarization surface charge density ρ_{sp} on the slab surface is 0.5C/m^2 . Find:

i) Polarization of the slab.

ii) Flux density in the slab.

iii) Flux density outside the slab (in air) .

iv) Field intensity inside and outside the slab.

Problem#5

The plane $x + 2y - 5z = 10$ separates the region which has $\mu_r = 2$ and on it $\vec{H} = 5\vec{a}_x + 6\vec{a}_y + 10\vec{a}_z$ from air. Find $\vec{H}$ in air.

Problem#6

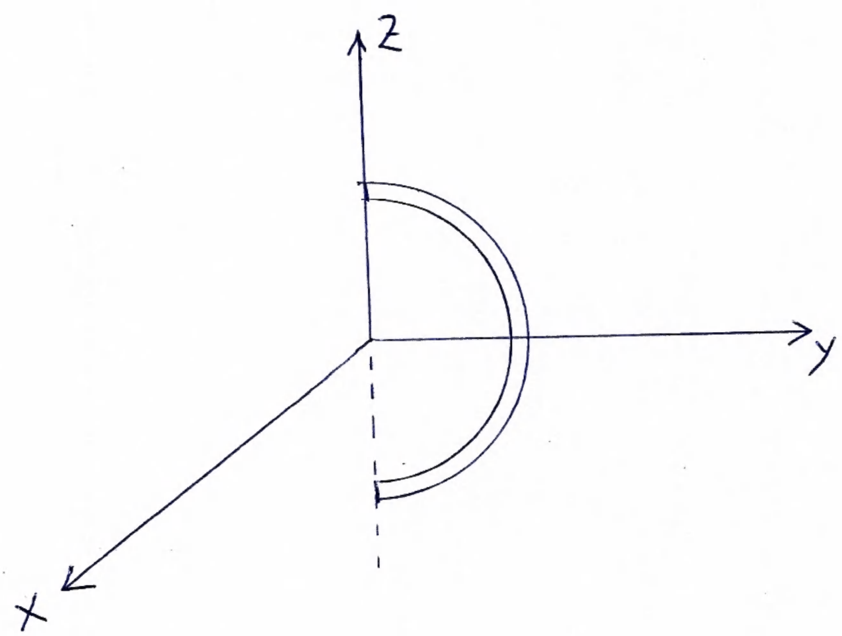
A plane wave at 100MHz is propagating in a lossy material. The phase of the electric field shifts 90^0 over a distance of 0.5m, and its peak value is reduced by 25% for each meter travelled. Find α , β , v_p.

Problem#7

At a certain frequency in copper ($\sigma = 58 \times 10^6\ S/m$) the phase constant is $3.71X10^5$ rad/m. Determine the frequency.

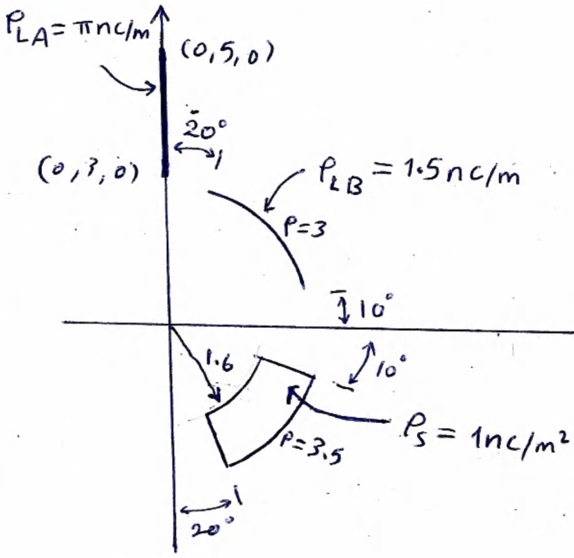
Problem#8

A semi-circular ring lying in the xy plane has a charge density $\rho_l = \rho_0 cos\theta\ C/m$, where θ is the angle measured from the z-axis as shown in the fig. Find $\vec{E}$ for points (x,0,0) along the x-axis.



Problem#9

The fig below shows three separate charge distributions in the z=0 plane in free space. Find the potential at P(0,0,6).

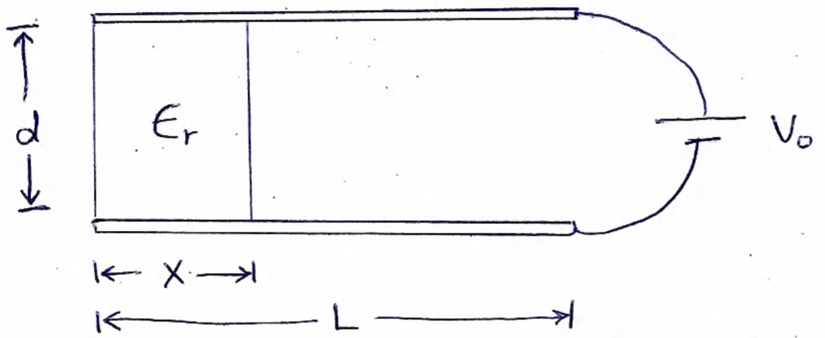


Problem#10

A parallel plate capacitor of width w, length L and separation d is partially filled with a dielectric medium of ϵ_r as shown in the fig. A battery of V_0 volts is connected between the plates.

i) Find $\vec{D}, \vec{E}$ and ρ_S in each region.

ii) Find distance x such that the electrostatic energy stored in each region is the same.



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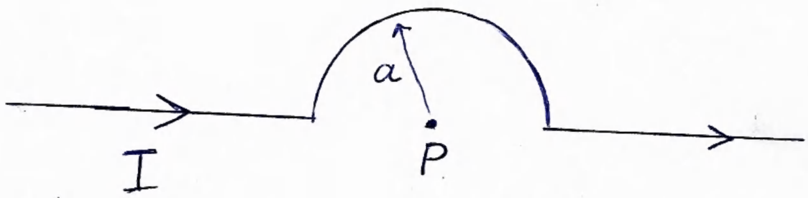
Problem#12

Let a filamentary current of 5mA be directed from infinity to the origin on the positive z-axis and then back out to infinity on the positive x-axis. Find $\vec{B}$ at P(0,1,0).

Problem#13

For the current shown. Find the magnetic field at the point P.



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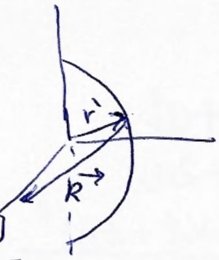
Q8) $\vec{r'} = \vec{a_y} R\sin\theta' + \vec{a_z} R\cos\theta'$

$\vec{R} = x\vec{a_x} - \vec{a_y} R\sin\theta' - \vec{a_z} R\cos\theta'$



$$E = \int_0^{\pi} \frac{\rho_e\, dl'}{4\pi |R|^2} \vec{a_R} = \int_0^{\pi} \frac{\rho_0 \cos\theta' R d\theta' \left[x\bar{a}_x - a_y R\sin\theta' - a_z R\cos\theta'\right]}{4\pi\epsilon_0 (x^2 + R^2\sin^2\theta' + R^2\cos^2\theta')^{3/2}}$$

$$= \frac{-\rho_0 R^2}{\epsilon_0 4\pi (x^2+R^2)^{3/2}} \int_0^{\pi} \cos^2\theta' d\theta'$$

$$= \frac{-\rho_0 R^2}{8\epsilon (x^2+R^2)^{3/2}} \vec{a_z}$$

Q9) ① due to ρ_{LA}

$$\phi = \int_3^5 \frac{\pi \times 10^{-9}\, dy'}{4\pi\epsilon_0 \sqrt{(y')^2+6^2}} = \frac{10^3}{4\times 8.854} \ln\left(y' + \sqrt{y'^2+6^2}\right)\Big|_3^5 = 7.83 \text{ V}.$$

② due to ρ_{LB}

$$\phi = \int_{\pi/18}^{7\pi/18} \frac{1.5\times 10^{-9} (3 d\phi')}{4\pi\epsilon_0 \sqrt{3^2+6^2}} = 6.03\left(\frac{7\pi}{18} - \frac{\pi}{18}\right) = 6.31 \text{ V}$$

③ due to ρ_s

$$\phi = \int_{\frac{\pi}{18}}^{\frac{7\pi}{18}} \int_{1.6}^{3.5} \frac{10^{-9} \rho' d\rho' d\phi'}{4\pi\epsilon_0 \sqrt{(\rho')^2+6^2}} = \frac{10^{-9}}{4\pi\epsilon_0} \left(\frac{\pi}{3}\right) \int_{1.6}^{3.5} \frac{\rho' d\rho'}{\sqrt{(\rho')^2+36}}$$

$$= \frac{10^{-9}}{4\pi\epsilon_0} \left(\frac{\pi}{3}\right) \sqrt{(\rho')^2+36}\Big|_{1.6}^{3.5} = 6.93 \text{ V}$$

ϕ at $P = 7.83 + 6.31 + 6.93 = 21.1$ V

Q10) $\vec{E} = -\vec{a_y} \frac{V_0}{d}$ in all space.

$\vec{D}(\text{air}) = -\vec{a_y} \frac{\epsilon_0 V_0}{d}$, $\vec{D}(\text{dielectric}) = -\vec{a_y} \frac{\epsilon_r \epsilon_0 V_0}{d}$

on top plate: $\rho_s(\text{air}) = \frac{\epsilon_0 V_0}{d}$, $\rho_s(\text{dielectric}) = \frac{\epsilon_r \epsilon_0 V_0}{d}$

$U = \frac{1}{2}\int \epsilon |E|^2 dv$

$$U_{\text{dielectric}} = \frac{1}{2} A\epsilon_r\epsilon_0 \int_0^x \frac{V_0^2}{d^2} dx = \frac{A\epsilon_r\epsilon_0 V_0^2}{2d^2} x$$

$$U_{\text{air}} = \frac{1}{2} A\epsilon_0 \int_x^L \frac{V_0^2}{d^2} dx = \frac{A\epsilon_0 V_0^2 (L-x)}{2d^2}$$

$U_{\text{dielectric}} = U_{\text{air}}$

$\epsilon_r x = L - x$

$x = \frac{L}{\epsilon_r + 1}$

Q1

a)

$$\bar{A} = \frac{\vec{a}_\rho}{\rho} \equiv \frac{\vec{a}_x \cos\phi + \vec{a}_y \sin\phi}{\sqrt{x^2+y^2}} = \frac{x}{x^2+y^2}\vec{a}_x + \frac{y}{x^2+y^2}\vec{a}_y$$ rectangular.

$$\bar{A} = \frac{r\sin\theta\cos\phi}{r^2\sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\phi}\left(\vec{a}_r \sin\theta\cos\phi + \bar{a}_\theta \cos\theta\cos\phi - \bar{a}_\phi \sin\phi\right)$$

$\cancel{\vec{a}_\rho \cdot \vec{a}_x = \cos\phi}$

$$+ \frac{r\sin\theta\sin\phi}{r^2\sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\phi}\left(\bar{a}_r \sin\theta\sin\phi + \bar{a}_\theta \cos\theta\sin\phi + \bar{a}_\phi \cos\phi\right)$$

$$= \frac{\cos\phi}{r\sin\theta}\left(\vec{a}_r \sin\theta\cos\phi + \vec{a}_\theta \cos\theta\cos\phi\right) + \frac{\sin\phi}{r\sin\theta}\left(\vec{a}_r \sin\theta\sin\phi + \bar{a}_\theta\cos\theta\sin\phi\right)$$

$$= \vec{a}_r\left(\frac{\cos^2\phi}{r} + \frac{\sin^2\phi}{r}\right) + \vec{a}_\theta\left(\cos^2\phi\frac{\cos\theta}{r\sin\theta} + \sin^2\phi\frac{\cos\theta}{r\sin\theta}\right)$$

$$= \frac{\vec{a}_r}{r} + \frac{\cot\theta}{r}\vec{a}_\theta$$

$$\oint A \cdot ds = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} a\sin\theta \, d\theta \, d\phi = 4\pi a$$

note: $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

b)

$$\oint \bar{A} \cdot d\bar{s} = \int_{-a}^{a}\int_{-a}^{a} \frac{a}{y^2+a^2} \, dy\,dz + \int_{-a}^{a}\int_{-a}^{a} \frac{a}{y^2+a^2} \, dy\,dz + \int_{-a}^{a}\int_{-a}^{a} \frac{a}{x^2+a^2} \, dx\,dz + \int_{-a}^{a}\int_{-a}^{a} \frac{a}{x^2+a^2} \, dx\,dz$$

$$= 8a\tan^{-1}\left(\frac{x}{a}\right)\Big|_{-a}^{a} = 8a\left(\tan^{-1}(1) - \tan^{-1}(-1)\right) = 8a\left(\frac{\pi}{2}\right) = 4\pi a.$$

Q12)

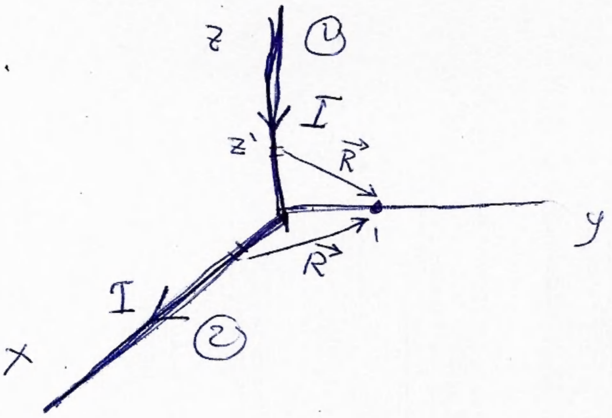
For ①

$\vec{R} = \vec{a_y} - z'\vec{a_z}$

$$B = \int_l \frac{\mu I d\vec{l'} \times \vec{a_R}}{4\pi |R|^2}$$

$$= \int_0^{\infty} \frac{-\mu I dz' \vec{a_z} \times (\vec{a_y} - z'\vec{a_z})}{4\pi (1+z'^2)^{3/2}}$$

$$= \vec{a_x} \frac{I\mu}{4\pi} \int_0^{\infty} \frac{dz'}{(1+z'^2)^{3/2}} = \vec{a_x} \frac{\mu_0 I}{4\pi} \frac{z'}{\sqrt{z'^2+1}} \Big|_0^{\infty} = \vec{a_x}\, 0.4\mu_0$$



Similarly for ②

$\vec{B} = \vec{a_z}\, 0.4\mu_0$

$\therefore \vec{B} = 0.4\mu_0 (\bar{a}_x + \bar{a}_z)$ ~~mA~~ mT

note:

$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a\sqrt{x^2+a^2}}$$